International Journal of Theoretical Physics, Vol. 24, No. 2, 1985

Calculation of the Strong Coupling, α_s, from Considerations of Virtual Synchrotron Radiation Resulting in Hadron Pair Emission

D. White[1]

Received November 9, 1983

Based upon a recently published work in which a model for hadron pair production via virtual synchrotron radiation in uniform magnetic fields was introduced, an ansatz for calculating α_s is presented. The resulting function is unique within the above-mentioned context. Along with a scheme introduced in which vector mesons arise by virtue of quark spin-flip with accompanying gluon emission, the α_s function is used to successfully explain the observed hadronic widths of the ρ, ϕ, and J mesons. In addition the hadronic and leptonic widths of the upsilon meson are calculated. Based upon gathered upsilon data, a charge on the b quark of $(2e/3)$ is found to be consistent with the above model. Finally, the derived α_s function is compared with results of gauge-theoretical calculations.

1. INTRODUCTION

In a recent article (White, 1982a) a QED-based model for production of hadron pairs (pions and kaons) via virtual synchrotron radiation (VSR) in uniform magnetic fields was introduced. The calculation of hadron pair production rates involves an ad hoc incorporation of a Lorentz amplitude function where appropriate to represent the resonant contribution at a given vector meson mass. The nonresonant contribution to the hadron pair production rates is provided by ordinary virtual photon mediated matrix elements associated with no spin flip of the synchrotron electron in making its transition from one Landau level to another. At the time of White's work (White,(1982a), only a few preliminary results of the VSR calculations were available. In Section 2 we will view the results of numerical computer computations associated with $\pi^+\pi^-$, $\pi^\circ\pi^\circ$, and $\mu^+\mu^-$ emission in which the synchrotron electron makes transitions to its ground state. The initial

[1]Department of Physics, Roosevelt University, 430 S. Michigan, Chicago, Illinois 60605.

0020-7748/85/0200-0201$04.50/0

states of the VSR electrons are picked to be principal quantum number $n = 1$, 1000, and 10^6, and for each state a wide range of initial energies is picked: from $1.00001 m_{\pi^+}c^2$ to 23 GeV $\cong 165 m_{\pi^+}c^2$, where $m_{\pi^+}c^2$ = charged pion rest energy = 139.57 MeV.

In Section 3 the results of the above calculations are employed in conjunction with the QCD theoretical prediction regarding the ratio of the cross section (Gasiorowicz and Rosher, 1981) $\sigma(e^+e^-)\to$ hadrons to $\sigma(e^+e^-)\to\mu^+\mu^-$ to calculate α_s. Specifically, we make the correspondence between a cross section in colliding beams and a rate (λ) of particle pair production in external fields, limit the hadronic production to pions and kaons, and use the formula

$$\frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)} \cong \frac{\lambda(\text{hadron pairs})}{\lambda(\text{muon pairs})} \cong 3\sum_{i=1}^{3} q_i^2\left(1+\frac{\alpha_s}{\pi}\right) \tag{1}$$

where i denotes flavors and q_i = ratio of quark charge to electron charge for each flavor [up (u): $q_1 = 2/3$; down (d): $q_2 = 1/3$; strange (s): $q_3 = 1/3$).

In Section 4 an ansatz based upon simple quantum mechanical principles generalized to incorporate gluon emission involving quark spin flip during hadronization is employed in order to describe vector mesons. We will find that the model presented not only is totally consistent with the previously ad hoc VSR methodology, but it also, when used in conjunction with the VSR-derived α_s function, accurately yields the hadronic widths of all of the vector mesons. A case will then be made for the necessity of the b quark charge to be $2e/3$.

In the final section a comparison of the VSR-α_s function is compared to same derived from consideratons of renormalization calculations within QCD gauge theories. We will find the functional forms for the two types of α_s functions to be identical but with differing coefficients. Implications of these findings will be discussed.

2. NUMERICAL CALCULATION OF VSR HADRON PAIR PRODUCTION RATES

In accord with previously presented techniques for calculating general VSR pair production rates involving electron–positron pairs (White, 1981), heavy lepton pairs (muons and tauons) (White, 1982b), and hadron pairs [pions and kaons (White, 1982a)], rates of production of $\mu^+\mu^-$, $\pi^\circ\pi^\circ$, and $\pi^+\pi^-$ pairs have been numerically calculated by computer[2] for a large number of physical conditions, all of which correspond to the VSR electron (assumed to be moving initially in a transverse uniform magnetic field, **H**)

[2]An IBM 4341 at University of Illinois, Chicago Circle Campus.

making a transition to its ground state, i.e., the state with principal quantum number, $n_f = 0$. The initial principal quantum numbers, n, picked for the electron are $n = 1$, 1000 (1 K), and 10^6 (1 M). For each n, the range of energies, E_n, for the electron is chosen to be from very near the $\pi^+\pi^-$ pair production threshold, viz., 1.00001 $(2m_{\pi^+}c^2) \cong 279.14$ MeV, to many times the threshold energy, viz., 82.38 $(2m_{\pi^+}c^2) \cong 23$ GeV.

In terms of the quantum mechanical critical field for electrons ($H_0 \cong 4.41 \times 10^{13}$ G), muons ($H_{0\mu} \cong 1.887 \times 10^{18}$ G), and pions ($H_{0\pi} \cong 3.293 \times 10^{18}$ G) the following range ($H_{\min}$ to $H_{\max}$) of magnetic field values for each of the three initial quantum numbers of the electron was employed:

For $n = 1$, $H_{\min} = 1.50 \times 10^5 H_0 = 3.50 H_{0\mu} = 2.01 H_{0\pi}$
and $H_{\max} = 1.01 \times 10^9 H_0 = 2.37 \times 10^4 H_{0\mu} = 1.36 \times 10^4 H_{0\pi}$.
For $n = 1$ K, $H_{\min} = 150 H_0 = 0.0035 H_{0\mu} = 0.0020 H_{0\pi}$
and $H_{\max} = 1.01 \times 10^6 H_0 = 23.68 H_{0\mu} = 13.57 H_{0\pi}$.
For $n = 1$ M, $H_{\min} = 0.15 H_0 = 3.50 \times 10^{-6} H_{0\mu} = 2.01 \times 10^{-6} H_{0\pi}$
and $H_{\max} = 1013 H_0 = 0.0237 H_{0\mu} = 0.0136 H_{0\pi}$.

Although the $H_{\max}$ for $n = 1$ is rather exotic,[3] the associated radiative corrections to the ground state energy[4] even at this intense field strength reaches only a relative magnitude of 27%. As the calculation is done in Born approximation, i.e., the emerging pair is treated as free, corrections due to the binding of the $\mu^+\mu^-$ and the $\pi^+\pi^-$ pair to the magnetic field would seem to become important for $H \gtrsim H_{0\mu}$ and $H \gtrsim H_{0\pi}$, respectively. Assuming such corrections to be on the order of $(\alpha/\pi)[\ln(H/H_{0\mu})]^2$ and $(\alpha/\pi)[\ln(H/H_{0\pi})]^2$, as are radiative corrections in the intense field range, the worst case ($n = 1$; muon pair production) would yield a relative correction on the order of 25%. In any case the reliability of the calculation for a particular initial electron energy, E_n, improves with increasing n to the point that for the worst case associated with $n = 1$ M, viz., $E_n = 23$ GeV and $H_{\max} = 1013 H_0$, radiative corrections are ~3% and $H_{\max}$ is much less than both $H_{0\mu}$ and $H_{0\pi}$. In addition, apart from radiative corrections, all $\pi^\circ\pi^\circ$ rates are valid since they are not bound to **H** upon emerging.

In Tables I–III are some results of the numerical calculations, the tolerance of same being approximately 1%. The energy, E_n, of the primary electron and the ratio of E_n to $2m_{\pi^+}c^2$ are listed along with the associated rates of production of $\mu^+\mu^-$, $\pi^\circ\pi^\circ$, and $\pi^+\pi^-$ pairs, $\lambda_{n0}^{(\mu^+\mu^-)}$, $\lambda_{n0}^{(\pi^\circ\pi^\circ)}$, and $\lambda_{n0}^{(\pi^+\pi^-)}$, respectively.

It is evident from the $\pi^+\pi^-$ results that near threshold, where $(E_n/2m_ic^2) \equiv \beta_i = 1 + \varepsilon_i$ with $\varepsilon_i \ll 1$ and i referring to a particle type with

[3] $H \sim 10^{20}$ G is expected upon theoretical grounds near instabilities on neutron stars. See Yu. Kovaleu (1980).

[4] Relative radiative corrections go as $(\alpha/4\pi)[\ln(2H/H_0)]^2$.

Table I

E_1 (GeV)	$(E_1/2m_{\pi^+}c^2)$	$\lambda_{10}^{(\mu^+\mu^-)}$ (s^{-1})	$\lambda_{10}^{(\pi^\circ\pi^\circ)}$ (s^{-1})	$\lambda_{10}^{(\pi^+\pi^-)}$ (s^{-1})
0.27914	1.00001	4.95×10^{16}	2.46×10^{14}	1.42×10^{7}
0.27917	1.0001	4.95×10^{16}	2.48×10^{14}	1.27×10^{9}
0.27942	1.001	4.99×10^{16}	2.63×10^{14}	1.33×10^{11}
0.282	1.010	5.37×10^{16}	4.49×10^{14}	1.75×10^{13}
0.307	1.098	9.87×10^{16}	5.64×10^{15}	3.07×10^{15}
0.358	1.28	2.27×10^{17}	3.79×10^{16}	3.14×10^{16}
0.409	1.46	3.97×10^{17}	9.80×10^{16}	9.01×10^{16}
0.460	1.65	5.99×10^{17}	1.82×10^{17}	1.76×10^{17}
0.511	1.83	8.30×10^{17}	2.87×10^{17}	2.87×10^{17}
0.562	2.01	1.09×10^{18}	4.11×10^{17}	4.19×10^{17}
0.613	2.20	1.36×10^{18}	5.50×10^{17}	5.74×10^{17}
0.715	2.56	1.97×10^{18}	8.69×10^{17}	9.52×10^{17}
0.818	2.93	2.65×10^{18}	1.24×10^{18}	1.46×10^{18}
0.920	3.30	3.37×10^{18}	1.64×10^{18}	2.07×10^{18}
1.022	3.66	4.15×10^{18}	2.08×10^{18}	2.72×10^{18}
1.278	4.58	6.27×10^{18}	3.30×10^{18}	4.41×10^{18}
1.533	5.49	8.59×10^{18}	4.67×10^{18}	6.21×10^{18}
2.044	7.32	1.37×10^{19}	7.74×10^{18}	1.01×10^{19}
2.555	9.15	1.93×10^{19}	1.12×10^{19}	1.42×10^{19}
5.110	18.31	5.19×10^{19}	3.15×10^{19}	3.78×10^{19}
10.22	36.61	1.28×10^{20}	8.04×10^{19}	9.28×10^{19}
23.00	82.38	3.46×10^{20}	2.18×10^{20}	2.46×10^{20}

$n = 1 \to 0$ pair production rates

mass m_i, the corresponding rate, $\lambda_{n0}^{(i)}$, is approximately proportional to ε_i^2. Consistent with this result is that

$$\lambda_{n0}^{(i)} \propto [\ln \beta_i]^2 \qquad \text{near threshold} \tag{2}$$

In the region of asymptotically high energies, however, the rates go as

$$\lambda_{n0}^{(i)} \propto \beta_i[\ln \beta_i] \qquad \text{for } \beta_i \gg 1 \tag{3}$$

which is a result commensurate with the asymptotic behavior of VSR e^+e^-, $\mu^+\mu^-$, and $\tau^+\tau^-$ pair production rates (see White, 1982b). Empirical asymptotic formulas for $\beta_i \gg 1$ have been found for the results of the numerical calculations previously discussed. We find

$$\lambda_{n0}^{(\mu^+\mu^-)} \to (\alpha/3\pi)\ln(E_n/2m_\mu c^2)\lambda_{n0} \qquad \text{for all } n \tag{4a}$$

$$\lambda_{10}^{(\pi^\circ\pi^\circ)} \to (2/3)(\alpha/3\pi)\ln(E_n/2m_{\pi^\circ}c^2)\lambda_{10} \tag{4b}$$

$$\lambda_{n0}^{(\pi^\circ\pi^\circ)} \to (1/2)(\alpha/3\pi)\ln(E_n/2m_{\pi^\circ}c^2)\lambda_{n0} \qquad \text{for } n \gg 1 \tag{4c}$$

Table II

E_{1K} (GeV)	$(E_{lK}/2m_{\pi^+}c^2)$	$\lambda_{1K0}^{(\mu^+\mu^-)}$ (s^{-1})	$\lambda_{1K0}^{(\pi^0\pi^0)}$ (s^{-1})	$\lambda_{1K0}^{(\pi^+\pi^-)}$ (s^{-1})
0.27914	1.00001	1.02×10^{14}	1.71×10^{12}	1.89×10^{5}
0.27917	1.0001	1.02×10^{14}	1.72×10^{12}	1.72×10^{7}
0.27942	1.001	1.02×10^{14}	1.80×10^{12}	1.73×10^{9}
0.282	1.010	1.08×10^{14}	2.74×10^{12}	1.80×10^{11}
0.307	1.098	1.66×10^{14}	1.74×10^{13}	1.15×10^{13}
0.358	1.28	3.05×10^{14}	6.53×10^{13}	5.82×10^{13}
0.409	1.46	4.64×10^{14}	1.27×10^{14}	1.21×10^{14}
0.460	1.65	6.37×10^{14}	1.97×10^{14}	1.96×10^{14}
0.511	1.83	8.24×10^{14}	2.76×10^{14}	2.82×10^{14}
0.562	2.01	1.02×10^{15}	3.60×10^{14}	3.77×10^{14}
0.613	2.20	1.23×10^{15}	4.50×10^{14}	4.85×10^{14}
0.715	2.56	1.68×10^{15}	6.44×10^{14}	7.62×10^{14}
0.818	2.93	2.15×10^{15}	8.54×10^{14}	1.20×10^{15}
0.920	3.30	2.66×10^{15}	1.08×10^{15}	1.58×10^{15}
1.022	3.66	3.19×10^{15}	1.32×10^{15}	1.91×10^{15}
1.278	4.58	4.61×10^{15}	1.96×10^{15}	2.72×10^{15}
1.533	5.49	6.13×10^{15}	2.65×10^{15}	3.58×10^{15}
2.044	7.32	9.45×10^{15}	4.17×10^{15}	5.42×10^{15}
2.555	9.15	1.30×10^{16}	5.83×10^{15}	7.39×10^{15}
5.110	18.31	3.35×10^{16}	1.54×10^{16}	1.84×10^{16}
10.22	36.61	8.08×10^{16}	3.78×10^{16}	4.36×10^{16}
23.00	82.38	2.13×10^{17}	1.01×10^{17}	1.14×10^{17}

$n=1$ K→0 pair production rates

$$\lambda_{10}^{(\pi^+\pi^-)} \to (2/3)(\alpha/3\pi)[\ln(E_n/2m_{\pi^+}c^2)+0.62]\lambda_{10} \tag{4d}$$

$$\lambda_{n0}^{(\pi^+\pi^-)} \to (1/2)(\alpha/3\pi)[\ln(E_n/2m_{\pi^+}c^2)+(3/5)]\lambda_{n0} \quad \text{for } n \gg 1 \tag{4e}$$

In the above formulas λ_{n0} refers to the ordinary synchrotron radiation rate for the corresponding electron initial and final conditions.

3. THE CALCULATION OF α_s

It is well known from gauge invariance theories that the ratio of cross section for hadron production form colliding e^+e^- beams $[\sigma(e^+e^- \to \text{hadrons})]$ to that of muon pair production $[\sigma(e^+e^- \to \mu^+\mu^-)]$ is given by (see Gasiorowicz and Rosher, 1981, equation (2.10, p. 963)

$$R = 2[1+(\alpha_s/\pi)] \tag{5}$$

Table III

E_{1M} (GeV)	$(E_{1M}/2m_{\pi^+}c^2)$	$\lambda_{1M0}^{(\mu^+\mu^-)}$ (s^{-1})	$\lambda_{1M0}^{(\pi^\circ\pi^\circ)}$ (s^{-1})	$\lambda_{1M0}^{(\pi^+\pi^-)}$ (s^{-1})
0.27914	1.00001	1.30×10^8	2.99×10^6	1.15
0.27917	1.0001	1.30×10^8	3.01×10^6	1.03×10^2
0.27942	1.001	1.32×10^8	3.17×10^6	8.82×10^3
0.282	1.010	1.57×10^8	5.12×10^6	5.35×10^5
0.307	1.098	6.57×10^8	7.48×10^7	5.04×10^7
0.358	1.28	5.18×10^9	1.15×10^9	1.02×10^9
0.409	1.46	2.03×10^{10}	5.67×10^9	5.40×10^9
0.460	1.65	5.35×10^{10}	1.68×10^{10}	1.67×10^{10}
0.511	1.83	1.10×10^{11}	3.74×10^{10}	3.79×10^{10}
0.562	2.01	1.93×10^{11}	6.88×10^{10}	7.16×10^{10}
0.613	2.20	3.02×10^{11}	1.12×10^{11}	1.19×10^{11}
0.715	2.56	5.94×10^{11}	2.30×10^{11}	2.71×10^{11}
0.818	2.93	9.68×10^{11}	3.87×10^{11}	5.39×10^{11}
0.920	3.30	1.41×10^{12}	5.74×10^{11}	8.33×10^{11}
1.022	3.66	1.90×10^{12}	7.87×10^{11}	1.13×10^{12}
1.278	4.58	3.28×10^{12}	1.40×10^{12}	1.93×10^{12}
1.533	5.49	4.82×10^{12}	2.09×10^{12}	2.80×10^{12}
2.044	7.32	8.17×10^{12}	3.62×10^{12}	4.68×10^{12}
2.555	9.15	1.18×10^{13}	5.29×10^{12}	6.67×10^{12}
5.110	18.31	3.22×10^{13}	1.48×10^{13}	1.77×10^{13}
10.22	36.61	7.88×10^{13}	3.69×10^{13}	4.25×10^{13}
23.00	82.38	2.08×10^{14}	9.86×10^{13}	1.11×10^{14}

$n = 1$ M$\rightarrow$0 pair production rates

in the region of asymptotically high energies and in the approximation that the masses of the muons and hadrons are considered as identical. The factor "2" on the right-hand side of equation (5) is consistent with limiting the final state hadrons to pions and kaons, and α_s, the strong coupling parameter, is treated as a function of energy. The term "(α_s/π)" represents the first-order gluon emission process during hadronization of vacuum quark/anti-quark formation via a virtual photon. Second- and higher-order terms in α_s are thought to become negligible in the high-energy regime.

In order to calculate α_s from considerations of VSR we substitute for the e^+e^- vertices relevant in equation (5) the ee' vertices involving VSR production of muons, pions, and kaons (the ee' notation signifying a primary electron in its initial state in a uniform magnetic field emitting a virtual photon while making a transition to its ground state). Hence the ratio takes the form

$$R_{\text{VSR}} \cong \frac{\lambda_{n0}^{(\pi^+\pi^-)} + \lambda_{n0}^{(\pi^\circ\pi^\circ)} + \lambda_{n0}^{(K^+K^-)} + \lambda_{n0}^{(K^\circ\bar{K}^\circ)}}{\lambda_{n0}^{(\mu^+\mu^-)}} \tag{6}$$

which is then set equal to the right-hand side of equation (5). We limit the VSR ratio to include only pion and kaon pairs for two principal reasons: First, the VSR model of hadron pair production is universally descriptive of nonresonant contributions to pion and kaon pair production rates in general. The vector mesons associated with hadron pairs (ρ, ϕ, J, Y) show up in VSR rate calculations as Lorentz amplitudes as a function of virtual photon energy to be incorporated in the virtual photon mass distribution which is then integrated over (see White, 1982a). Therefore, if all Lorentz amplitudes of all the above vector mesons are incorporated in the VSR calculations, all hadronic states associated with hadron pairs in the final state are in fact included. Hence R_{VSR} takes the form

$$R_{\text{VSR}} \cong \frac{\lambda_{n0}^{(\text{all pion+kaon pairs})}}{\lambda_{n0}^{(\mu^+\mu^-)}} \cong 2\left[1+\left(\frac{\alpha_s}{\pi}\right)\right] \tag{7}$$

and the ratio includes terms descriptive of ρ, ϕ, J, and Y formation. What has been neglected has been the consideration of the formation of VSR-compatible hadronic states eventuating in three or more pions or kaons in the final state, such as the ω-meson which decays predominantly into $\pi^+\pi^-\pi^\circ$. We are assuming that rates for producing yet additional hadrons (beyond a pair) are suppressed significantly relative to pair production rates. Since at least one additional photon to quark/antiquark coupling would be required, the production rates for three or more hadrons would be of order $\alpha \cong (137.036)^{-1}$ or less times the pair production rate under similar physical conditions. The latter statement constitutes the basic argument behind the second reason for including only the rates for pion and kaon production via VSR in calculating R_{VSR}. In actuality only the ρ-meson is included in what follows because compared to the ρ, the effects of the ϕ, J, etc. have negligible influence on the evaluation of R_{VSR} for the following reasons: The contribution to the VSR rate by a vector meson resonance is proportional to the area between the virtual photon mass distribution and the nonresonant background of same (see White, 1982a). Hence, relative to the ρ, the contributions of other vector mesons are proportional to essentially the ratio of their width (Γ_v) to that of the $\rho(\Gamma_\rho)$ and to the square of charge of the type of quark making up the vector meson.

For example, if we define $\lambda(V)$ to be the resonant contribution to the VSR rate (at very high energies) due to a vector meson of type V (i.e., $V=\rho$, ϕ, J, etc.), we find

$$\lambda(\phi) \cong \lambda(\rho)(\Gamma_\phi/\Gamma_\rho)q_s^2 \tag{8a}$$

where q_s = strange quark charge = 1/3, $\Gamma_\phi \simeq 4$ MeV, and $\Gamma_\rho \cong 150$ MeV. Inserting the appropriate numbers yields

$$\lambda(\phi) \cong (1/337)\lambda(\rho) \tag{8b}$$

For the J we obtain

$$\lambda(J) \cong \lambda(\rho)(\Gamma_J/\Gamma_\rho)q_c^2 \tag{9a}$$

where q_c = charm (c) quark charge = 1/3 and $\Gamma(J) \simeq 0.060$ MeV, yielding

$$\lambda(J) \cong (1/22500)\lambda(\rho) \tag{9b}$$

Preliminary Y data suggests $\lambda(\Upsilon) < \lambda(J)$.

Keeping in mind that in our defining equation for α_s [equation (5)] the physical conditions must correspond to the asymptotically free region of high energies where the mass differences can be neglected, we must pick VSR conditions correspondingly. We therefore choose $n \to \infty$, in which case $E_n \to \infty$ and $H \to 0$, and $n_f = 0$, in which case all available initial energy goes into the hadronization process. In such limits the VSR and colliding beam situations are as similar as they can be.

In order to minimize the effect of mass differences in our calculation we will use equations (4a, c, e) but will renormalize the rates by dividing out the overall mass dependence factor scaling each of the rates, viz., $\ln(E_n/2m_ic^2)$. In addition, owing to the negligible influence of the ϕ on the KK description, we assume

$$\lambda_{n0}^{(K^+K^-)} \to (1/2)(\alpha/3\pi)\ln(E_n/2m_{K^+}c^2) \qquad \text{for } n \gg 1 \tag{10a}$$

and

$$\lambda_{n0}^{(K^\circ\bar{K}^\circ)} \to (1/2)(\alpha/3\pi)\ln(E_n/2m_{K^\circ}c^2) \qquad \text{for } n \gg 1 \tag{10b}$$

thus making the renormalized $\pi^\circ\pi^\circ$, $K^\circ\bar{K}^\circ$, and K^+K^- rates identical. Denoting the renormalized rates by $\bar{\lambda}_{n0}^{(i)}$ we obtain for $n \to \infty$ ($E_n \to \infty$; $H \to 0$):

$$\bar{\lambda}_{n0}^{(\mu^+\mu^-)} = (\alpha/3\pi)\lambda_{n0} \tag{11a}$$

$$\bar{\lambda}_{n0}^{(\pi^\circ\pi^\circ)} = \lambda_{n0}^{(K^\circ\bar{K}^\circ)} = \lambda_{n0}^{(K^+K^-)} = (1/2)(\alpha/3\pi)\lambda_{n0} \tag{11b}$$

and

$$\bar{\lambda}_{n0}^{(\pi^+\pi^-)} = \frac{1}{2}\frac{\alpha}{3\pi}\left[1 + \frac{3}{5\ln(E_n/2m_{\pi^+}c^2)}\right] \tag{11c}$$

We thus find

$$R_{\text{VSR}} \cong (3/2) + 1/2\{1 + (3/5)[\ln(E_n/2m_{\pi^+}c^2)]^{-1}\} \tag{12}$$

Setting R_{VSR} equal to the right-hand side of equation (5) yields

$$3/2 + 1/2\{1 + (3/5)[\ln(E_n/2m_{\pi^+}c^2)]^{-1}\} \cong 2\left(1 + \frac{\alpha_s}{\pi}\right) \tag{13a}$$

or

$$2+(3/10)[\ln(E_n/2m_{\pi^+}c^2)]^{-1} \cong 2+\frac{2\alpha_s}{\pi} \tag{13b}$$

Solution of equation (13b) yields for α_s,

$$\alpha_s = (3\pi/20)[\ln(E_n/2m_{\pi^+}c^2)]^{-1} \tag{14}$$

The above result is quite compatible with experimental values of α_s at the J energy (Gasiorowicz and Rosher, 1981; Close, 1979), viz., $\alpha_s|_{E_n=3095\text{ MeV}} \cong 0.20$ in that equation (14) yields for $E_n = 3095$ MeV

$$\alpha_s = 0.1959 \qquad \text{(at the } J \text{ energy)} \tag{15}$$

We will find in the next section that when equation (14) is employed in conjunction with a simple quantum mechanical model involving quark gluon emission, the α_s function derived is very successful in explaining major properties of the vector mesons (ρ, ϕ, J, and Υ).

4. A QUANTUM-MECHANICAL MODEL FOR VECTOR MESON CONTRIBUTIONS IN VSR: QUARK SPIN FLIP VIA GLUON EMISSION

The Lorentz amplitudes for the vector mesons,

$$\mathscr{L}_v = \frac{\Gamma_v^2}{4(q-q_0)^2+\Gamma_v^2} \tag{16}$$

where $q \equiv$ virtual photon energy and $q_0 \equiv$ vector meson rest energy, become part of the VSR description of hadron pair production in a simple and natural way in terms of the following ansatz for the general process:

During hadronization one of the quarks making up one of the pions (or kaons) in the pair production process flips its spin at certain preferred energies (the rest renergies of the vector mesons). In the spin flip process the associated quark emits a gluon whose energy is absorbed by the VSR virtual photon (γ_v). In this picture the gluon (G) couples directly to γ_v with unit probability so that in terms of a formal QED description of the VSR process not involving vector mesons (nonresonant contribution), viz.,

$$W_0 = \int d^4k|J_v|^2 g_v \tag{17}$$

where $g_v = 1$ for $v = \rho$ and $g_v = 1/9$ for $v = \phi$ or J, and where $|J_v|^2$ incorporates the entire VSR nonresonant amplitude (see Ref. 1, equation 2.8), the inclusion of vector mesons yields

$$W = W_0 + \int\int d^4k\, d^4Q |J_v^*|^2 \delta(k-Q) g_v \mathscr{L}_v + \text{cross terms} \qquad (18)$$

where $|J_v^*|^2$ represents the VSR spin flip matrix elements, $\int d^4k$ signifies summation over γ_v indices, and $\int d^4Q$ signifies summation over G indices ($Q \equiv$ gluon 4-wave vector). In equation (18) the term $\mathscr{L}_v$ arises from the gluon's natural line width by direct analogy to photon line widths associated with a given transition between two energy levels. (In most QED contexts, consideration of photon line widths is of no consequence, but in the VSR context involving hadrons, it is the gluon line width which carries all the information regarding the α_s calculation.)

As an independent check of the above ansatz (and of the α_s calculation as well), we borrow from QED the formal quantum mechanical description of a general photon emission process and make the appropriate substitutions so as to be descriptive of the gluon emission process alone (apart from VSR considerations). Specifically, in all quantum systems in which natural decay occurs between an excited level and a ground state level, the absorption cross section goes as[5]

$$\sigma(\omega) = \frac{\alpha}{2\pi} |V|^2 \left(\frac{1}{m}\right)^2 \frac{1}{\omega} \mathscr{L}(\omega) \qquad (19)$$

where $\mathscr{L}(\omega)$ is a Lorentz amplitude with a peak at some $\omega = \omega_0$ and with a width Γ. The term $|V|^2$ represents the square of the matrix element descriptive of the photon-emission process, and the system has mass m.

We may use equation (19) to predict the width of vector mesons "standing alone," i.e., after all nonresonant background events have been subtracted if we make the following substitutions to take us from a general QED process to a specific QCD process:

(1) We substitute for the photon frequency ω the gluon energy Q_0.

(2) We evaluate the right hand side of equation (19) at a specific vector meson mass, m_v, i.e., $Q_0 = m \equiv m_v$. Hence the associated Lorentz amplitude equals unity.

(3) We require $|V|^2$ be proportional to $\sum_i (q_i)^4$ where $q_i \equiv$ quark charge magnitude (in units of e) associated with the quarks comprising the relevant vector meson.

The above criterion is consistent with a spin-spin interaction proportional to q_i^2 giving rise to spin-flip transitions, and the sum is required only in the case of the ρ, which is comprised of both the u and d quarks. Hence,

[5]See Merzbacher, 1970, p. 486. We choose to use the coefficient "$\alpha/2\pi$" in equation (19). We do so without loss of generality since $|V|^2$ is treated as an unknown constant in what follows.

we postulate in the case of the ρ, there are two spin-flip contributions: one due to a $u\bar{u}$ interaction involving a spin flip and one due to a $d\bar{d}$ interaction involving same. Hence $|V|^2 \propto q_u^4 + q_d^4$ for the ρ.

(4) We postulate $|V|^2$ to be proportional only to $\sum_i (q_i)^4$, i.e.,

$$|V|^2 = \left[\sum_i (q_i)^4\right] |\tilde{V}|^2 \tag{20}$$

where $|\tilde{V}|^2$ is a constant, i.e., the precise form of the interaction is universal to all vector mesons except for quark charge differences.

(5) We replace α by α_s as derived in Section 3. Hence, for each vector meson width calculation we employ

$$\alpha_s(V) = \frac{3\pi}{20}\left[\ln\left(\frac{m_v}{2m_{\pi^+}}\right)\right]^{-1} \tag{21}$$

Accordingly we find in terms of the above ansatz

$$\Gamma_v \propto \left(\frac{\alpha_s(V)}{2\pi}\right)\left(\frac{1}{m_v^3}\right)|\tilde{V}|^2\left(\sum_i (q_i)^4\right) \tag{22}$$

Normalizing to the ρ where $\sum_i (q_i)^4 = 17/81$ we may calculate $|\tilde{V}|^2$. Accordingly,[6]

$$\Gamma_\rho = 152 \text{ MeV} = \frac{\alpha_s(\rho)}{2\pi}|\tilde{V}|^2 \frac{17}{81} \tag{23}$$

Since from equation (21),

$$\alpha_s(\rho) = 0.4626, \tag{24}$$

$$152 \text{ MeV} = \frac{0.4626}{2\pi}|\tilde{V}|^2 \frac{17}{81} \tag{25}$$

yielding

$$|\tilde{V}|^2 = 9838 \text{ MeV} \tag{26}$$

Hence, for any vector meson we find

$$\Gamma_v = \left(\frac{\alpha_s(V)}{2\pi}\right)\left(\frac{m_\rho}{m_v}\right)^3 |\tilde{V}|^2 \sum_i q_i^4 \tag{27}$$

where $|\tilde{V}|^2$ is given by equation (26).

[6]All meson data in equations (23)-(28d) are taken from the meson table listings of April 1976, published in the 58th edition of the *Handbook of Chemistry and Physics*, R. C. Weast, ed. (CRC Press, Cleveland, 1977), pp. F-270-272.

The test of the foregoing procedure consists of finding vector meson widths on the basis of equation (17) and comparing the results to experimental findings as to the hadronic widths involving hadron pairs as decay products.

For the ϕ we find from (27)

$$\Gamma_\phi = \frac{\alpha_s(\phi)}{2\pi}\left(\frac{m_\rho}{m_\phi}\right)^3 |\tilde{V}|^2 q_s^4, \quad \text{or} \tag{28a}$$

$$\Gamma_\phi = \frac{0.3637}{2\pi}\left(\frac{773}{1020}\right)^3 (9838)\frac{1}{81}, \quad \text{yielding} \tag{28b}$$

$$\Gamma_\phi = 3.06 \text{ MeV} \tag{28c}$$

which is to be compared against the total KK width from experiment:

$$\Gamma_\phi(\exp) = (3.3 \pm 0.3) \text{ MeV} \tag{28d}$$

For the J we find from (4.12)

$$\Gamma_J = \frac{\alpha_s(J)}{2\pi}\left(\frac{m_\rho}{m_J}\right)^3 |\tilde{V}|^2 q_c^4, \quad \text{or} \tag{29a}$$

$$\Gamma_J = \frac{0.1959}{2\pi}\left(\frac{773}{3095}\right)^3 (9838)\frac{1}{81}, \quad \text{yielding} \tag{29b}$$

$$\Gamma_J = 59 \text{ keV} \tag{29c}$$

which is to be compared against the total hadronic width from experiment:

$$\Gamma_J(\exp) = (58 \pm 12) \text{ keV} \tag{29d}$$

The charge of the b quark is not known with certainty at this time, nor is the upsilon width, while rudimentary knowledge of its leptonic width, $\Gamma_{\Upsilon(e^+e^-)}$, has been accumulated (see Gasiorowicz and Rosher, 1981; also, *CRC Handbook*, 63rd ed.), viz., $\Gamma_{\Upsilon(e^+e^-)} \cong 1.5$ keV. Employing equation (27) and assuming $q_b = 1/3$, we obtain [since $\alpha_s(\Upsilon) = 0.1338$]

$$\Gamma_{\Upsilon 1} = 1.4 \text{ keV} \cong \Gamma_{\Upsilon(e^+e^-)}, \tag{30}$$

thus suggesting that $q_b = 1/3$ is incorrect within the context of the present model. If $q_b = 2/3$, on the other hand, we obtain

$$\Gamma_{\Upsilon 2} = 23 \text{ keV} \tag{31}$$

The associated leptonic width would then be

$$\Gamma_{\Upsilon(e^+e^-)} \cong \frac{\alpha}{\alpha_s(\Upsilon)}\Gamma_{\Upsilon 2} \cong 1.3 \text{ keV} \tag{32}$$

in fair agreement with experiment. On the strength of the success of the model as it pertains to the ρ, ϕ, and J, we would therefore expect future experiments to show that $q_b = 2/3$ and $\Gamma_\Upsilon \cong 23$ keV = hadronic width of the Υ.

In concluding this section we note that the VSR-α_s function found in Section 3 used in conjunction with the gluon emission model of the present section appears to be quite precise in its determination of hadronic widths of vector mesons and is quite possibly a useful tool in the determination of the charge of the b quark and others beyond. In the next and last section we will view how the α_s of equation (14) compares to α_s calculated from general QCD gauge invariance theories.

5. COMPARISON OF THE α_s FUNCTION BASED UPON THE VSR-GLUON EMISSION MODEL (VSR-GEM) AND α_s BASED UPON GAUGE INVARIANCE THEORIES IN QCD

Based upon a QCD calculation of vacuum polarization within the context of gauge invariance theories the α_s function is given by [Gasiorowicz and Rosher, 1981, equation (2.9), p. 963]

$$\alpha_s(\text{QCD}) = \frac{6\pi}{33 - 2N}\left(\ln \frac{E_n}{\Lambda}\right)^{-1} \tag{33}$$

where N = total number of different quark types (flavors) and Λ is a parameter to be determined by experiment. As an outgrowth of the above calculation, there are now many scheme-dependent theories within QCD aimed at predicting the correct value of Λ. Most schemes place Λ in the range between 0.10 GeV and 0.50 GeV (Tung and Johnson, private communication).

Based upon the VSR-GEM we find

$$\alpha_s(\text{VSR}) = \frac{6\pi}{40}\left[\ln\left(\frac{E_n}{2m_{\pi^+}c^2}\right)\right]^{-1} \tag{34}$$

We thus find α_s(VSR) identical in form to α_s(QCD) but with different coefficients of the reciprocal of the logarithmic function. Of interest is that in the VSR-GEM $2m_{\pi^+}c^2$ takes the place of the Λ parameter. As such, the VSR-GEM uniquely determines Λ, i.e.,

$$\Lambda(\text{VSR}) = 2m_{\pi^+}c^2 \cong 0.28 \text{ GeV} \tag{35}$$

a value which lies near the median of most of the QCD theoretical predictions.

The above-mentioned coefficients, however, viz.,

$$C(\mathrm{QCD})=\frac{6\pi}{33-2N} \qquad \text{in QCD} \tag{36a}$$

and

$$C(\mathrm{VSR})=\frac{6\pi}{40} \qquad \text{in VSR-GEM} \tag{36b}$$

are seriously discrepant. For $N=6$ or 7, for example,

$$C(\mathrm{QCD})\cong 2C(\mathrm{VSR}) \tag{37}$$

The one loop calculations used to derive equation (36a) (the factor "33" stemming from the gluon loop and the factor "$2N$" stemming from the quark/antiquark loop), however, are not trustworthy owing to the fact that α_s increases with decreasing energy (Gasiorowicz and Rosher, 1981, p. 963). Total agreement could be reached if, given that no modifications in the gluon loop calculation are necessary, the term "$-2N$" in equation (36a) were replaced by "$+N$," in which case $N=7$ would represent exact equivalence and $N=6$ or 8 would represent choices possible within a reasonable margin of error (±2.5%).

What leads to the QCD determination of α_s is a renormalization scheme of quarks and gluons in analogy to the renormalization scheme of electrons and photons. Nevertheless, treating QCD as separate from QED and proceeding by analogy leads to a free parameter (Λ). Within VSR-GEM, leptons, hadrons, vector mesons, gluons, and quarks all exist within one unified structure. Since VSR is just a special case within QED, it is certainly possible to construct other QED-GEM frameworks for other contexts. Whether all such constructions will yield a unique $\alpha_s(\mathrm{QED})=\alpha_s(\mathrm{VSR})$ is a matter of conjecture, but of interest now is that it is simply not necessary to renormalize quarks and gluons within VSR-GEM. The latter statement follows because although $\alpha_s(\mathrm{VSR})\to\infty$ at $E_n\to 2m_{\pi^+}c^2$ (from above), actual VSR rates for hadronic processes in this limit go to zero [recall equation (2)]. It may therefore be true that since QED is already renormalized, when quarks and gluons are included as they are in VSR-GEM, no further modifications are necessary. [Recall the $\delta(k-Q)$ factor in equation (18) and that all quark charges are expressed in units of e.]

In closing we remark that the $\alpha_s(\mathrm{VSR})$ function [equation (34)] was derived in the limit of $E_n\to\infty$, $H\to 0$, and $n_f=0$, which corresponds completely to the limit of asymptotic freedom in QCD gauge theories. It is interesting to note that for $n=1$, $E_1\to\infty$, and $n_f=0$ which corresponds to the most severely constrained VSR situation possible for a given energy, we find [see equation (4a, b, d)] an α_s-VSR function associated with $n = 1$ and asymptotically high energies given by

$$\alpha_s^{(1)} = \frac{\pi}{3}\left(1 + \frac{0.62}{\ln(E_1/2m_{\pi^+}c^2)}\right) \tag{38}$$

Hence

$$\alpha_s^{(1)} \to \pi/3 \text{ as } E_1 \to \infty. \tag{39}$$

It is thus evident that α_s(VSR) is descriptive of asymptotic freedom only when $n \to \infty$ and $E_n \to \infty$ and that $\alpha_s^{(n)}$ approaches some finite constant $A_n \leq \pi/3$ for finite n in the limit of infinite E_n. The physical implications of the above result are that the widths of the vector mesons become greater with decreasing n in systems where hadronic VSR processes are taking place. The ρ width, for example, in the $n = 1$ case (neglecting radiative and binding corrections) would become

$$\Gamma_\rho(1) \cong \Gamma_\rho(\alpha_s^{(1)}(\rho)/\alpha_s(\rho)) \tag{40a}$$

or

$$\Gamma_\rho(1) \cong (152 \text{ MeV}) \frac{1.6846}{0.4626} \cong 554 \text{ MeV} \tag{40b}$$

or about 3.6 times larger than the result associated with asymptotic freedom. Such findings could be of major import for theories concerned with hadronic processes on pulsars, but we will leave a detailed discussion of $\alpha_s^{(n)}$ to a future article.

ACKNOWLEDGMENTS

I wish to thank Prof. Thomas Erber for his input in our many discussions on subjects pertaining to the present paper. I wish to thank Prof. Wu Ki Tung and Prof. Porter Johnson, both colleagues of Prof. Erber at the Illinois Institute of Technology, for their help in discussions regarding the Λ parameter. I also thank Roosevelt University for providing computer time for the research associated with the present paper and the Goldenberg Foundation for providing computer terminal equipment necessary for utilizing the IBM computer at Chicago Circle Campus.

REFERENCES

Close, F. E. (1979). *An Introduction to Quarks and Partons* (Academic, New York), p. 357.
Gasiorowicz, S., and Rosher, J. L. (1981). *Am. J. Phys.*, **49**, 954.

Kovaleu, Yu. (1980). *Astrophys. Space Sci.*, **67**, 387.
Merzbacher, E. (1970). *Quantum Mechanics* (Wiley, New York).
Tsai, W. Y., and Yildiz, A. (1973). *Phys. Rev., D* **8**, 3446.
Tung, Wu Ki, and Johnson, Porter (Illinois Institute of Technology), private communication.
White, D. (1981). *Phys. Rev., D* **24**, 526.
White, D. (1982a). *Phys. Rev., D* **26**, 3009.
White, D. (1982b). *Phys. Rev., D* **26**, 2924.